Homework #3 – Due Thursday, Oct. 5
COR1-GB.1305 – Statistics and Data Analysis

Problem 1

A space agency estimates that the chance of a "critical-item failure" (that is, a catastrophic failure) in their space shuttle's main engines is 2 in 126 for each mission.

(a) What is the probability that at least one of the six shuttle missions scheduled in the next two years results in a critical-item failure?

(b) What is the probability that at least one of the 10 shuttle missions scheduled over the next three years results in a critical-item failure?

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Problem 2

A survey of workers in the two plants of a manufacturing firm includes the question "How effective is management in responding to legitimate grievances of workers?" In plant 1, 48 of 192 workers respond "poor"; in plant 2, 80 of 248 workers respond "poor". An employee of the manufacturing firm is to be selected randomly.

Let A be the event "worker comes from plant 1" and let B be the event "response is poor."

(a) Find $P(A)$, $P(B)$, and $P(A \mid B)$.

(b) Are the events A and B independent?

(c) Find $P(B \mid A)$ and $P(B \mid A^c)$. Are they equal?

(d) Show that $P(B^c) \neq P(B^c \mid A^c)$.

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Problem 3 Empirical Rule for Sum of Two Dice

Suppose that you throw two dice. Each die can come up as 1, 2, 3, 4, 5 or 6, and the results from the two dice are independent of each other. We are interested in the random variable X, the sum of the two numbers that land face up. The possible values for X are $2, 3, \ldots, 12$.

(a) Make a table giving the probability distribution of X. Explain briefly how you did the calculations.

(b) Show that $E(X) = 7$ and $\text{var}(X) = 210/36 = 5.833$

(c) Although the distribution of X is not a normal distribution, a graph of it would look somewhat bell-shaped. (This is not a coincidence. The more dice you toss, the closer the distribution of the sum comes to a normal distribution. More on this later in the course.) For now, let's see how well the empirical rule works. Show that the probability that the z-score for X is between -1 and 1 is $24/36 = 0.667$. Show that the probability that the z-score for X is between -2 and 2 is $34/36 = 0.944$.

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Problem 4 Roulette Doubling (Martingale) System

Roulette wheels in casinos in the US have 38 numbers, of which two are green (0 and 00), 18 are black and 18 are red. A bet on black pays at even money, $1:1$ odds (though these odds are clearly not fair due to the green numbers.) Each of the 38 numbers is equally likely to occur on any given spin of the wheel, and results from successive spins are independent. (Casinos expend considerable effort to ensure that these properties hold; otherwise, gamblers would have opportunities for arbitrage.)

Let's consider a doubling system, at a table with a \$100 minimum bet, and a \$100,000 maximum bet. In terms of \$100 chips, that's a 1-chip minimum and a 1000-chip maximum. To start the system, bet 1 chip on Black. If you win, you're up \$100 and that's the end of the system. If you lose, bet 2 chips on Black. If you win at this point, you're once again up \$100 (having lost one chip and then won two chips), and that's the end of the system. If you lose, bet 4 chips on Black. Continue doubling if you lose. Once you finally win (no matter how long this takes), you will be up \$100, since successive powers of 2 add up to one less than the next power of two, for example, $1 + 2 + 4 + 8 = 15 = 16 - 1$. So as long as you can keep playing until the first time Black is rolled, you will win your \$100.

So, what's the catch? Unfortunately, the table maximum of 1000 chips eventually becomes a problem, since if you lose 10 times in a row, you will be down $1 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256 + 512 = 1023$ chips, and you will not be allowed to make the next bet, since a 1024-chip bet would exceed the table limit.

Suppose that you play this system just once, until either you get your \$100 profit, or you spectacularly go bust with a losing streak of 10 non-Black numbers. Compute the expected net winnings (profit minus loss) for the system, in dollars.

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