

Hypothesis Tests – Solutions
COR1-GB.1305 – Statistics and Data Analysis

Introduction

1. An analyst claims to have a reliable model for Yahoo’s quarterly revenues. His model predicted that the most recent quarterly revenues could be described as a normal random variable with mean \$1.5B and standard deviation \$0.1B. In actuality, the revenues were \$1.0B. Is there evidence of a problem with the analyst’s model? Why or why not?

Solution: The observed revenues were $(1.0 - 1.5)/(0.1) = 5$ standard deviations away from the expected value predicted by the analyst’s model. If the analyst’s model were correct, the chance of observing a deviation this large or larger would be extremely small (0.00005733%). This is evidence that there is a problem with the analyst’s model.

2. Prof. Perry has a coin, which he claims to be fair (50% chance of “heads,” and 50% chance of “tails”). He flips the coin 10 times, and gets “heads” all 10 times. Do you believe him that the coin is fair? Why or why not?

Solution: If the coin were fair, there would be a 0.2% chance of getting the same outcome on all 10 flips. That is, the observed string of 10 heads in a row would be extremely unlikely. This is evidence that the coin is not fair.

Test on a Population Mean

3. (Adapted from Stine and Foster, 4M 16.2). Does stock in IBM return a different amount on average than T-Bills? We will attempt to answer this question by using a dataset of the 264 monthly returns from IBM between 1990 and 2011. Over this period, the mean of the monthly IBM returns was 1.26% and the standard deviation was 8.27%. We will take as given that the expected monthly returns from investing in T-Bills is 0.3%.

- (a) What is the sample? What are the sample mean and standard deviation?

Solution: The $n = 264$ monthly IBM returns from 1990 to 2011. The sample mean and standard deviation (in %) are

$$\bar{x} = 1.26$$

$$s = 8.27$$

- (b) What is the relevant population? What are the interpretations of population mean and standard deviation?

Solution: All monthly IBM returns (past, present, and future). The population mean, μ represents the expected return for a month in the future. The population standard deviation, σ , represents the standard deviation of the monthly returns for all months (past, present, and future).

- (c) What are the null and alternative hypotheses for testing whether or not IBM gives a different expected return from T-Bills (0.3%)?

Solution:

$$H_0 : \mu = 0.3$$

$$H_a : \mu \neq 0.3$$

- (d) Use an appropriate test statistic to summarize the evidence against the null hypothesis.

Solution: If the null hypothesis were true ($\mu = 0.3$), then the sample mean would have been a normal random variable with mean $\mu_{\bar{X}} = 0.3$ and standard deviation $\sigma_{\bar{X}} = \sigma/\sqrt{n}$. The test statistic

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

would follow a t distribution with $n - 1 = 263$ degrees of freedom. The observed value of this statistic is

$$t = \frac{1.26 - 0.3}{8.27/\sqrt{264}} = 1.886$$

- (e) If the null hypothesis were true (there were no difference in expected monthly returns between IBM and T-Bills) what would be the chance of observing data at least as extreme as observed?

Solution: If we approximate the distribution of the test statistic under H_0 as a standard normal random variable, then the chance of observing data at least as extreme as observed would be

$$p = P(|Z| > 1.886) \approx 0.05743.$$

- (f) Is there compelling evidence (at significance level 5%) of a difference in expected monthly returns between IBM and T-Bills?

Solution: No, since $p \geq 0.05$, there is not compelling evidence.

- (g) What assumptions do you need for the test to be valid? Are these assumptions plausible?

Solution: Since $n \geq 30$, we do not need to assume that the population is normal. We need that the observed sample is a simple random sample from the population; this might not hold if the period under observation (1990–2011) is not typical for IBM.

Test Statistic and Observed Significance Level (p -value)

4. In each of the following examples, for the hypothesis test with

$$H_0 : \mu = \mu_0$$

$$H_a : \mu \neq \mu_0$$

find the test statistic (t) and the p -value.

(a) $\mu_0 = 5$; $\bar{x} = 7$; $s = 10$; $n = 36$.

Solution:

$$\begin{aligned} t &= \frac{7 - 5}{10/\sqrt{36}} \\ &= 1.2 \\ p &\approx P(|Z| > 1.2) \\ &= 0.2301 \end{aligned}$$

(b) $\mu_0 = 90$; $\bar{x} = 50$; $s = 200$; $n = 64$.

Solution:

$$\begin{aligned} t &= \frac{50 - 90}{200/\sqrt{64}} \\ &= -1.6 \\ p &\approx P(|Z| > 1.6) \\ &= 0.1096 \end{aligned}$$

(c) $\mu_0 = 50$; $\bar{x} = 49.4$; $s = 2$; $n = 100$.

Solution:

$$\begin{aligned} t &= \frac{49.4 - 50}{2/\sqrt{100}} \\ &= -3 \\ p &\approx P(|Z| > 3) \\ &= 0.002700 \end{aligned}$$

5. For each example from problem 4:

(a) Indicate whether a level 5% test would reject H_0 .

Solution: We reject H_0 if $p < 0.05$: (a) do not reject H_0 ; (b) do not reject H_0 ; (c) reject H_0 .

(b) Indicate whether a level 1% test would reject H_0 .

Solution: We reject H_0 if $p < 0.01$: (a) do not reject H_0 ; (b) do not reject H_0 ; (c) reject H_0 .