Random Variables
COR1-GB.1305 – Statistics and Data Analysis

Bayes' Rule

1. Every year in March there is a standardized exam for people who want to be licensed sheep herders. It happens that, with probability 0.45, a person will pass this exam. In the process of screening people, it turns out that among those who passed the exam, 60% had taken college courses in biology. It happens also that 30% of all those who take the exam had college courses in biology. Find the probability that a person with college courses in biology will pass the exam.

2. Amazon.com maintains a list of all registered customers, along with their email addresses. During July, they sent coupons to 20% of their customers. They recorded that 5% of their customers made purchases in July, and 40% of all purchases were made with coupons. In this problem we will compute the proportion of customers sent a coupon in July who made a purchase in that month. For simplicity, we will assume that customers either make 0 or 1 purchases in July.

   (a) Consider a random customer, and define two events:

   $$\text{Coupon} = \text{the customer received a coupon in July},$$
   $$\text{Purchase} = \text{the customer made a purchase in July}.$$

   Express all percentages given in the problem statement as probabilities or conditional probabilities of these two events. Example: P(Coupon) = 0.20.

   (b) Use Bayes' rule to compute the proportion of custoers sent a coupon in July who made a purchase that month.

Probability Distribution Function and Expectation

3. Consider the following game:

    1. You pay \$6 to flip a coin.
    2. If the coin lands heads, you get \$10; otherwise, you get nothing.

    (a) Would you play this game? Why or why not?

    (b) What is the random experiment involved in the game? What are the sample space? What are the probabilities of the sample points?

    (c) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the value of W for each of the sample points.

    (d) Describe W in terms of its probability distribution function (PDF).

    (e) What are your expected winnings? That is, what is μ, the expectation of W?

4. Suppose you flip two coins. Let X be the random variable which counts the number of heads on the two tosses.
   (a) List all of the sample points of the experiment, along with the corresponding values of X.

   (b) Compute the probability distribution function of X.

   (c) Compute the expectation of X.

   (d) What is the interpretation of the expectation of X?

5. Let X be a random variable describing the number of cups of coffee a randomly-chosen member of the class drinks on a typical day. There is a 22% chance that the student has one cup, a 16% chance that the student has two cups, a 16% chance that the student has three cups, an 11% chance that the student has four cups, and a 3% chance that the student has five cups. Also, there is a 32% chance that the student doesn't drink any coffee.
   (a) Let $p(x)$ be the probability distribution function of X. Fill in the following table:

x	0	1	2	3	4	5
$p(x)$						

   (b) Find $\mathrm{E}(X)$, the expectation of X.

   (c) What is the interpretation of the expectation of X?

Variance and Standard Deviation

6. This is a continuation of problem 5.
    (a) Find var(X) and sd(X), the variance and standard deviation of X, the number of cups of coffee that a random student from the class drinks on a typical day.

    (b) What is the interpretation of the standard deviation of X?

7. Consider the following game:
    1. You pay \$6 to pick a card from a standard 52-card deck.
    2. If the card is a diamond ($\diamondsuit$), you get \$22; if the card is a heart ($\heartsuit$), you get \$6; otherwise, you get nothing.

    Perform the following calculations to decide whether or not you would play this game.

    (a) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the PDF of W.

    (b) What are your expected winnings? That is, what is μ, the expectation of W?

    (c) What is the standard deviation of W?

    (d) What are the interpretations of the expectation and standard deviation of W?