Regression Inference and Forecasting – Solutions
COR1-GB.1305 – Statistics and Data Analysis

Inference

1. Here are the least squares estimates from the fitting the model

$$\text{Price} = \beta_0 + \beta_1 \text{Size} + \varepsilon_1$$

for $n = 18$ apartments in Greenwich Village. Price is measured in units of \$1000 and size is measured in units of 100 ft^2.

```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
101.375  86.87%     86.05%      81.13%

Coefficients

Term              Coef  SE Coef  T-Value  P-Value   VIF
Constant         182.3     62.4     2.92    0.010
Size(100sqft)    44.95     4.37    10.29    0.000  1.00

Regression Equation

Price($1000) = 182.3 + 44.95 Size(100sqft)
```

(a) Construct a 95% confidence interval for β_1.

> **Solution:** We use
> $$\hat{\beta}_1 \pm t_{\alpha/2}\text{SE}(\hat{\beta}_1),$$
> where $\alpha = .05$ and we have $n - 2 = 16$ degrees of freedom. This gives
> $$44.95 \pm 2.120(4.37) = 44.95 \pm 9.26,$$
> or $(35.69, 54.21)$.

(b) What is the meaning of the confidence interval for β_1?

> **Solution:** We are 95% confident that if we increase size by 100 square feet, then mean price will increase by an amount between \$35.7K and \$54.2K.

(c) What is the meaning of a 95% confidence interval for β_0? In the context of the housing data, is this useful?

> **Solution:** This would be a confidence interval for the mean price of apartments with size 0. This is nonsensical (no apartments have size 0), and thus not useful.

(d) Perform a hypothesis test at level 5% of whether or not the is a linear relationship between Size and mean Price.

Solution: We are interested in the following null and alternative hypotheses:

$$H_0 : \beta_1 = 0 \quad \text{(no linear relationship)}$$
$$H_a : \beta_1 \neq 0 \quad \text{(linear relationship)}$$

Based on the Minitab output, the p-value for this test is below 0.001. Thus, we reject the null hypothesis at level 5%. There is a statistically significant linear relationship between size and mean price.

We can also do this problem using a rejection region. We reject H_0 at level α if $|T| > t_{\alpha/2}$, where

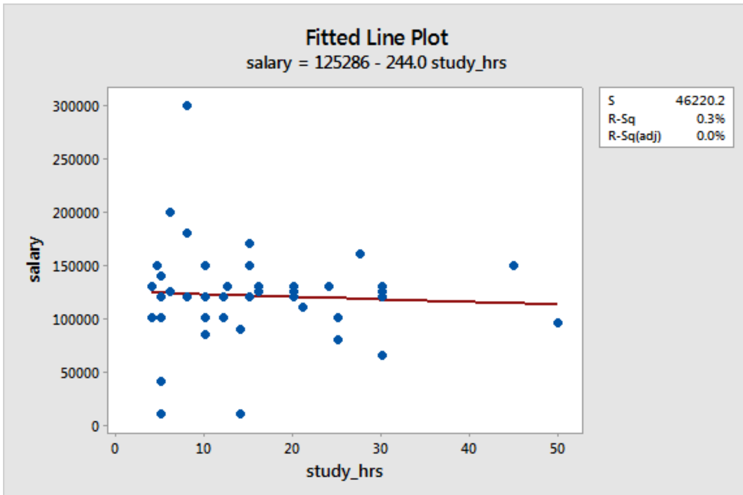
$$T = \frac{\hat{\beta}_1 - 0}{\text{SE}(\hat{\beta}_1)} = \frac{44.95}{4.37} = 10.286$$

For level $\alpha = .05$, we have $t_{\alpha/2} = t_{.025} = 2.120$ (using $n - 2 = 16$ degrees of freedom). Since $|T| > 2.120$, we reject H_0.

2. 45 students reported their planned weekly study time (in hours) and their expected starting salaries (in dollars). We will use this data to examine the relationship between these two variables. We fit the model

$$\text{Salary} = \beta_0 + \beta_1 \text{StudyHrs} + \varepsilon$$

using least-squares. The scatterplot at Minitab regression output follow.




```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
46220.2  0.32%      0.00%       0.00%

Coefficients

Term          Coef  SE Coef  T-Value  P-Value   VIF
Constant    125286    12493    10.03    0.000
study_hrs     -244      658    -0.37    0.713  1.00

Regression Equation

salary = 125286 -244 study_hrs
```

(a) Quantify the relationship between expected salary and planned study time using a 95% confidence interval. (You will need the value $t_{.025,43} \approx 2.021$.)

Solution: A 95% confidence interval for β_1 is

$$\begin{aligned}\hat{\beta}_1 \pm t_{.025,n-2}\text{se}(\hat{\beta}_1) &= (-244) \pm (2.021)(658)\\ &= -244 \pm 1330\\ &= (-1574, 1086)\end{aligned}$$

We can be 95% confident that increasing planned study time by 1 hour is associated with changing expected starting salary by an amount between -1574 dollars and 1086 dollars.

(b) Perform a hypothesis test to determine if there is a significant linear relationship between expected salary and planned study time.

Solution: The null and alternative hypotheses are

$$
\begin{aligned}
H_0 &: \beta_1 = 0 \qquad \text{(no linear relationship)} \\
H_a &: \beta_1 \neq 0 \qquad \text{(linear relationship)}
\end{aligned}
$$

The p-value for this test is $p = 0.713$. Since $p \geq 0.05$, we do not reject H_0; there is no significant relationship between expected salary and planned study time.

Forecasting

3. We used the regression model fit to the housing data to predict price at size 2000 ft^2:

```
Regression Equation

Price($1000) = 182.3 + 44.95 Size(100sqft)


Variable        Setting
Size(100sqft)        20


    Fit   SE Fit        95% CI               95% PI
1081.27  38.1287  (1000.44, 1162.10)  (851.667, 1310.88)
```

(a) Find a 95% confidence interval for the mean price of all apartments with size 2000 ft^2.

> **Solution:** This is given in the output: (1000.4, 1162.1). We 95% confidence, the mean price of all apartments with size 2000 ft^2 is between \$1,000,400 and \$1,152,100.

(b) Find a 95% prediction interval for the price of a particular apartments with size 2000 ft^2.

> **Solution:** Again, this is given in the output: (851.7, 1310.9). If someone tells us that a particular apartment has size 2000 ft^2, then we can say with 95% confidence that the price of the apartment is between \$851,700 and \$1,310,900.

(c) Make a statement about the prices of 95% of all apartments with size 2000 ft^2.

> **Solution:** To make a statement about *all* apartments, we use a prediction interval. With 95% confidence, 95% of all apartments with size 2000 ft^2 have sizes between \$851,700 and \$1,310,900.

(d) What is the difference between the confidence interval and the prediction interval?

> **Solution:** A confidence interval is a statement about the mean value of Y; a prediction interval is a statement about a particular value of Y (equivalently, all values of Y).

4. We fit a regression model to the 294 restaurants from the 2003 Zagat data. Our predictor variable is food quality (1–30), and our response variable is price ($). Here is the result of using the fitted model to predict the price when the food quality is 25.

```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
12.5559  27.93%     27.68%      26.86%

Coefficients

Term       Coef  SE Coef  T-Value  P-Value   VIF
Constant  -4.74     3.95    -1.20    0.232
Food      2.129    0.200    10.64    0.000  1.00

Regression Equation

Price = -4.74 + 2.129 Food

Variable  Setting
Food           25

    Fit   SE Fit        95% CI              95% PI
48.4832  1.33906  (45.8478, 51.1187)  (23.6315, 73.3349)
```

(a) What is the interpretation of the 95% confidence interval?

Solution: We are 95% confident that the average price of all 2003 New York City restaurants with quality ratings of 25 is between $45.84 and $51.12.

(b) What is the interpretation of the 95% prediction interval?

Solution: Approximately 95% of all 2003 New York City restaurants with quality ratings of 25 have prices between $23.63 and $73.34.

(c) Explain how the confidence interval is related to `Fit`, `SE Fit`, and `S`.

Solution: The 95% confidence interval for $\mathrm{E}(Y \mid x = 25)$ is approximately equal to

$$\hat{y}(25) \pm 2\mathrm{se}(\hat{y}(25)) = 48.4832 \pm (2)(1.33906).$$

(For an exact equivalence, use $t_{.025,n-2}$ instead of 2.)

(d) Explain how the prediction interval is related to `Fit`, `SE Fit`, and `S`.

Solution: The 95% prediction interval is approximately equal to

$$\hat{y}(25) \pm 2s = 48.4832 \pm (2)(12.5559).$$

(For an exact equivalence you would use the formula

$$\hat{y}(x) \pm t_{.025,n-2}\sqrt{s^2 + [\mathrm{se}\{\hat{y}(x)\}]^2};$$

you are not expected to know this formula.)