FORECASTING HOMEWORK 7

Recall that $\{x_t\}$ is a martingale if $E[x_{n+h} | x_n, x_{n-1}, \cdots] = x_n$ for all n and for all lead times $h > 0$. Actually, to establish that $\{x_t\}$ is a martingale, one simply needs to prove the above formula for $h = 1$ since it can be shown that if it holds for $h = 1$ it must hold for all $h > 0$.

1) Suppose $x_t = x_{t-1} + \varepsilon_t$ where $\varepsilon_t = e_t + \beta e_{t-1} e_{t-2}$, $\beta \neq 0$, and $\{e_t\}$ is strict white noise.

a) What is the best linear predictor of x_{n+1} based on $x_n, x_{n-1}, \cdots$? Justify your answer.

b) What is the best possible predictor of x_{n+1} based on $x_n, x_{n-1}, \cdots$? Justify your answer.

c) Compare your answers to a) and b) to decide whether $\{x_t\}$ is a martingale. (Keep in mind the discussion at the top of this handout).

2) Suppose $x_t = \alpha x_{t-1} + e_t$ where $\{e_t\}$ is strict white noise.

a) If $|\alpha| < 1$, is $\{x_t\}$ a martingale? Justify your answer.

b) If $\alpha = 1$, is $\{x_t\}$ a martingale? Justify your answer.

3) Suppose $\{\varepsilon_t\}$ are martingale differences. Suppose we "integrate" $\{\varepsilon_t\}$ to obtain a series $\{y_t\}$. Specifically, define $y_1 = \varepsilon_1$, $y_2 = \varepsilon_1 + \varepsilon_2$, etcetera.

a) Show that $y_t = y_{t-1} + \varepsilon_t$.

b) Use the results of a) to show that $\{y_t\}$ is a martingale. (Thus, integrating a martingale difference yields a martingale.)