

## The Central Limit Theorem

COR1-GB.1305 – Statistics and Data Analysis

1. Consider the population of all Fortune 500 CEOs and their salaries. Suppose that the mean salary (in millions of dollars) is  $\mu = 20$ , and the standard deviation of the salaries is  $\sigma = 5$ . You sample 50 CEOs and find their salaries.

(a) Draw a histogram of what you think the population looks like.

(b) Consider the sample mean  $\bar{X}$  to be a random variable. What is the expectation of  $\bar{X}$ ?

(c) What is the standard deviation of  $\bar{X}$ ?

(d) Draw a picture of what you think the PDF of  $\bar{X}$  looks like.

2. You draw a random sample of size  $n = 64$  from a population with mean  $\mu = 50$  and standard deviation  $\sigma = 16$ . From this, you compute the sample mean,  $\bar{X}$ .

(a) What are the expectation and standard deviation of  $\bar{X}$ ?

(b) Approximately what is the probability that the sample mean is above 54?

(c) Do you need any additional assumptions for part (c) to be true?

3. You draw a random sample of size  $n = 16$  from a population with mean  $\mu = 100$  and standard deviation  $\sigma = 20$ . From this, you compute the sample mean,  $\bar{X}$ .

(a) What are the expectation and standard deviation of  $\bar{X}$ ?

(b) Approximately what is the probability that the sample mean is between 95 and 105?

(c) Do you need any additional assumptions for part (c) to be true?