Hypothesis Tests – Solutions

COR1-GB.1305 – Statistics and Data Analysis

Introduction

1. An analyst claims to have a reliable model for Twitter's quarterly revenues. His model predicted that the most recent quarterly revenues could be described as a normal random variable with mean \$1.5B and standard deviation \$0.1B. In actuality, the revenues were \$1.0B. Is there evidence of a problem with the analyst's model? Why or why not?

   Solution: The observed revenues were $(1.0 - 1.5)/(0.1) = 5$ standard deviations away from the expected value predicted by the analyst's model. If the analyst's model were correct, the chance of observing a deviation this large or larger would be extremely small (0.00005733%). This is evidence that there is a problem with the analyst's model.

2. Prof. Perry has a coin, which he claims to be fair (50% chance of "heads," and 50% chance of "tails"). He flips the coin 10 times, and gets "heads" all 10 times. Do you believe him that the coin is fair? Why or why not?

   Solution: If the coin were fair, there would be a 0.2% chance of getting the same outcome on all 10 flips. That is, the observed string of 10 heads in a row would be extremely unlikely. This is evidence that the coin is not fair.

Test on a Population Mean

3. (Adapted from Stine and Foster, 4M 16.2). Does stock in IBM return a different amount on average than T-Bills? We will attempt to answer this question by using a dataset of the 264 monthly returns from IBM between 1990 and 2011. Over this period, the mean of the monthly IBM returns was 1.26% and the standard deviation was 8.27%. We will take as given that the expected monthly returns from investing in T-Bills is 0.3%.

   (a) What is the sample? What are the sample mean and standard deviation?

   Solution: The $n = 264$ monthly IBM returns from 1990 to 2011. The sample mean and standard deviation (in %) are

   $$\bar{x} = 1.26$$
   $$s = 8.27$$

   (b) What is the relevant population? What are the interpretations of population mean and standard deviation?

   Solution: All monthly IBM returns (past, present, and future). The population mean, μ represents the expected return for a month in the future. The population standard deviation, σ, represents the standard deviation of the monthly returns for all months (past, present, and future).

   (c) What are the null and alternative hypotheses for testing whether or not IBM gives a different expected return from T-Bills (0.3%)?

   Solution:

   $$H_0 : \mu = 0.3$$
   $$H_a : \mu \neq 0.3$$

(d) Use an appropriate test statistic to summarize the evidence against the null hypothesis.

Solution: If the null hypothesis were true ($\mu = 0.3$), then the sample mean would have been a normal random variable with mean $\mu_{\bar{X}} = 0.3$ and standard deviation $\sigma_{\bar{X}} = \sigma/\sqrt{n}$. The test statistic

$$T = \frac{\bar{X} - \mu_0}{S/\sqrt{n}}$$

would follow a t distribution with $n - 1 = 263$ degrees of freedom. The observed value of this statistic is

$$t = \frac{1.26 - 0.3}{8.27/\sqrt{264}} = 1.886$$

(e) If the null hypothesis were true (there were no difference in expected monthly returns between IBM and T-Bills) what would be the chance of observing data at least as extreme as observed?

Solution: If we approximate the distribution of the test statistic under H_0 as a standard normal random variable, then the chance of observing data at least as extreme as observed would be

$$p = P(|Z| > 1.886) \approx 0.05743.$$

(f) Is there compelling evidence (at significance level 5%) of a difference in expected monthly returns between IBM and T-Bills?

Solution: No, since $p \geq 0.05$, there is not compelling evidence.

(g) What assumptions do you need for the test to be valid? Are these assumptions plausible?

Solution: Since $n \geq 30$, we do not need to assume that the population is normal. We need that the observed sample is a simple random sample from the population; this might not hold if the period under observation (1990–2011) is not typical for IBM.

Test Statistic and Observed Significance Level (p-value)

4. In each of the following examples, for the hypothesis test with

$$H_0 : \mu = \mu_0$$
$$H_a : \mu \neq \mu_0$$

find the test statistic (t) and the p-value.

(a) $\mu_0 = 5$; $\bar{x} = 7$; $s = 10$; $n = 36$.

Solution:

$$\begin{aligned} t &= \frac{7-5}{10/\sqrt{36}} \\ &= 1.2 \\ p &\approx P(|Z| > 1.2) \\ &= 0.2301 \end{aligned}$$

(b) $\mu_0 = 90$; $\bar{x} = 50$; $s = 200$; $n = 64$.

Solution:

$$\begin{aligned} t &= \frac{50-90}{200/\sqrt{64}} \\ &= -1.6 \\ p &\approx P(|Z| > 1.6) \\ &= 0.1096 \end{aligned}$$

(c) $\mu_0 = 50$; $\bar{x} = 49.4$; $s = 2$; $n = 100$.

Solution:

$$\begin{aligned} t &= \frac{49.5-50}{2/\sqrt{100}} \\ &= -3p && \approx P(|Z| > 3) \\ &= 0.002700 \end{aligned}$$

5. For each example from problem 4:

(a) Indicate whether a level 5% test would reject H_0.

> **Solution:** We reject H_0 if $p < 0.05$: (a) do not reject H_0; (b) do not reject H_0; (c) reject H_0.

(b) Indicate whether a level 1% test would reject H_0.

> **Solution:** We reject H_0 if $p < 0.01$: (a) do not reject H_0; (b) do not reject H_0; (c) reject H_0.