

Probability Distribution Function and Expectation

1. Consider the following game:
 1. You pay \$6 to flip a coin.
 2. If the coin lands heads, you get \$10; otherwise, you get nothing.
- (a) Would you play this game? Why or why not?
- (b) What is the random experiment involved in the game? What are the sample space? What are the probabilities of the sample points?
- (c) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the value of W for each of the sample points.
- (d) Describe W in terms of its probability distribution function (PDF).
- (e) What are your expected winnings? That is, what is μ , the expectation of W ?

2. Suppose you flip two coins. Let X be the random variable which counts the number of heads on the two tosses.

(a) List all of the sample points of the experiment, along with the corresponding values of X .

(b) Compute the probability distribution function of X .

(c) Compute the expectation of X .

(d) What is the interpretation of the expectation of X ?

3. Let X be a random variable describing the number of cups of coffee a randomly-chosen member of the class drinks on a typical day. There is a 22% chance that the student has one cup, a 16% chance that the student has two cups, a 16% chance that the student has three cups, an 11% chance that the student has four cups, and a 3% chance that the student has five cups. Also, there is a 32% chance that the student doesn't drink any coffee.

(a) Let $p(x)$ be the probability distribution function of X . Fill in the following table:

x	0	1	2	3	4	5
$p(x)$						

(b) Find $E(X)$, the expectation of X .

(c) What is the interpretation of the expectation of X ?

Variance and Standard Deviation

4. This is a continuation of problem 3.

(a) Find $\text{var}(X)$ and $\text{sd}(X)$, the variance and standard deviation of X , the number of cups of coffee that a random student from the class drinks on a typical day.

(b) What is the interpretation of the standard deviation of X ?

5. Consider the following game:

1. You pay \$6 to pick a card from a standard 52-card deck.
2. If the card is a diamond (\diamond), you get \$22; if the card is a heart (\heartsuit), you get \$6; otherwise, you get nothing.

Perform the following calculations to decide whether or not you would play this game.

(a) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the PDF of W .

(b) What are your expected winnings? That is, what is μ , the expectation of W ?

(c) What is the standard deviation of W ?

(d) What are the interpretations of the expectation and standard deviation of W ?

Properties of Expectation

6. **Affine Transformations.** Let X be a random variable with expectation $\mu_X = 2$. What is the expectation of $5X + 2$?
7. **Sums of Independent Random Variables.** Let X and Y be random variables with $\mu_X = 1$, $\mu_Y = -5$. What is $E(X + Y)$?
8. Let X and Y be random variables with $\mu_X = -2$, $\mu_Y = 3$.
- (a) Find the expectation of $-3X + 2$.

 - (b) Find the expectation of $X + Y$.
9. You invite four people to go out to dinner on Friday night. The attendance probabilities for the four potential guests are 50%, 20%, 30%, and 90%.
- (a) Find the expected number of guests.

 - (b) The dinner will be a *prix fixe* meal, costing \$50 per person. What is the expected total cost for yourself and your guests?

 - (c) What is the interpretation of your answer to part (b)?