Introduction to Confidence Intervals

STAT-UB.0103 – Statistics for Business Control and Regression Models

The Central Limit Theorem (Review)

1. You draw a random sample of size $n = 64$ from a population with mean $\mu = 50$ and standard deviation $\sigma = 16$. From this, you compute the sample mean, $\bar{X}$.
   (a) What are the expectation and standard deviation of $\bar{X}$?

   (b) Approximately what is the probability that the sample mean is above 54?

   (c) Do you need any additional assumptions for part (c) to be true?

2. You draw a random sample of size $n = 16$ from a population with mean $\mu = 100$ and standard deviation $\sigma = 20$. From this, you compute the sample mean, $\bar{X}$.
   (a) What are the expectation and standard deviation of $\bar{X}$?

   (b) Approximately what is the probability that the sample mean is between 95 and 105?

   (c) Do you need any additional assumptions for part (c) to be true?

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3. Consider the following game. Population with mean μ and and known standard deviation $\sigma = 7$. I know μ, but you don't. You sample $n = 49$ observations from the population and compute the sample mean $\bar{X}$. Your goal is to guess the value of μ. Suppose you observe the sample mean $\bar{x} = 4.110$.

   (a) If μ were equal to 4, would $\bar{x} = 4.110$ be typical? Take "typical" to mean "we would observe a value like this about 95% of the time."

   (b) If μ were equal to 5, would $\bar{x} = 4.110$ be typical?

   (c) If μ were equal to 10, would $\bar{x} = 4.110$ be typical?

   (d) What is the largest value of μ for which a sample of $\bar{x} = 4.110$ would be considered typical?

   (e) What is the smallest value of μ for which a sample of $\bar{x} = 4.110$ would be considered typical?

   (f) What can you say about the random interval $(\bar{X} - 2, \bar{X} + 2)$?

   (g) What can you say about the observed interval $(\bar{x} - 2, \bar{x} + 2)$, where $x = 4.110$?

Confidence Intervals for a Population Mean (Known Variance)

4. A random sample of n measurements was selected from a population with unknown mean μ and known standard deviation σ. Calculate a 95% confidence interval for μ for each of the following situations:

   (a) $n = 49$, $\bar{x} = 28$, $\sigma = 28$

   (b) $n = 36$, $\bar{x} = 12$, $\sigma = 18$

   (c) $n = 100$, $\bar{x} = 125$, $\sigma = 50$

   (d) Is the assumption that the underlying population of measurements is normally distributed necessary to ensure the validity of the confidence intervals in parts (a)–(c)?

5. Complete the previous problem, with 99% confidence intervals instead of 95% confidence intervals.

6. Find the values of α and $z_{\alpha/2}$ for computing 99.9% confidence intervals. (If you don't have a z table, draw a bell curve with a shaded region showing the relationship between α and $z_{\alpha/2}$).

7. Find the values of α and $z_{\alpha/2}$ for computing 80% confidence intervals.