More Confidence Intervals – Solutions

STAT-UB.0103 – Statistics for Business Control and Regression Models

Population Mean (Unknown Variance)

1. Use the following sample means and sample standard deviations from the class survey to form 95% confidence intervals the population mean for each variable.

   (a) SAT score: $\bar{x} = 2160$, $s = 140$, $n = 57$.

   > **Solution:**
   >
   > $$\begin{aligned} 2160 \pm 2\frac{140}{\sqrt{57}} &= 2160 \pm 2 \cdot 18.5 \\ &= 2160 \pm 37 \end{aligned}$$

   (b) Hours spent studying each week: $\bar{x} = 16.6$, $s = 10.1$, $n = 60$.

   > **Solution:**
   >
   > $$\begin{aligned} 16.6 \pm 2\frac{10.1}{\sqrt{60}} &= 16.6 \pm 2 \cdot 1.3 \\ &= 16.6 \pm 2.6 \end{aligned}$$

   (c) Hours spent working each week (if employed): $\bar{x} = 10.3$, $s = 3.6$, $n = 13$

   > **Solution:**
   >
   > $$\begin{aligned} 10.3 \pm 2.179\frac{3.6}{\sqrt{13}} &= 10.3 \pm 2.179 \cdot 1.00 \\ &= 10.3 \pm 2.2 \end{aligned}$$

2. In problem 1, what are the relevant populations?

   > **Solution:** In (a) and (b): all NYU Stern freshmen and sophomores. In (c): all NYU Stern freshmen and sophomores who work.

3. In problem 1, what assumptions do we need for the confidence intervals to be valid? How could we check these assumptions?

   > **Solution:** In all cases, we need that the samples are unbiased. In (c), since $n < 30$, we need that the population is approximately normal. We can check the normality assumption by looking at a histogram. We cannot check the unbiasedness assumption.

Population Proportion

4. Use the following data from the class survey to estimate the relevant population proportions. Give 95% confidence intervals for these proportions.

   (a) Gender: 26 Female, 38 Male.

   Solution: We estimate the proportion of Female students in the population.

   $$\hat{p} = \frac{26}{26+38} = .40$$
   $$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.40)(.60)}{64}} = .06$$

   The 95% confidence interval is $.40 \pm 2 \cdot (.06) = .40 \pm .12$.

   (b) Employed: 51 No, 13 Yes.

   Solution: We estimate the proportion of employed students in the population.

   $$\hat{p} = \frac{13}{13+51} = .20$$
   $$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.20)(.80)}{64}} = .05$$

   The 95% confidence interval is $.20 \pm 2 \cdot (.05) = .20 \pm .10$.

   (c) Finance Major: 32 Yes, 31 No.

   Solution: We estimate the proportion of Finance majors in the population.

   $$\hat{p} = \frac{32}{32+31} = .51$$
   $$\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = \sqrt{\frac{(.51)(.49)}{63}} = .06$$

   The 95% confidence interval is $.51 \pm 2 \cdot (.06) = .51 \pm .12$.

5. In problem 4, what are the relevant populations?

Solution: All NYU Stern freshmen and sophomores.

6. In problem 4, what assumptions do we need for the confidence intervals to be valid? How could we check these assumptions?

Solution: In all cases, we need that the samples are unbiased. According to the book, we should have at least 15 Positive and 15 Negative observations in each case for the central limit theorem to apply. This holds for (a) and (c); it almost holds for (b). (Note: it is impossible for the population to be approximately normal in these cases since the variable is categorical/qualitative.)

Additional Confidence Interval Problems

7. The *Minneapolis Star Tribune* (August 12, 2008) reported that 73% of Americans say that Starbucks coffee is overpriced. The source of this information was a national telephone survey of 1,000 American adults conducted by Rasmussen Reports. Find and interpret a 95% confidence interval for the population proportion.

Solution: We use the same formula as in the previous problem, but now $\hat{p} = .73$ and $n = 1000$. Thus,

$$\sqrt{\hat{p}(1-\hat{p})/n} = .014.$$

The 95% confidence interval is

$$.73 \pm .028.$$

8. Researchers recorded expenses per full-time equivalent employee for each in a sample of 1,751 army hospitals. The sample yielded the following summary statistics: $\bar{x} = \$6,563$ and $s = \$2,484$. Estimate the mean expenses per full-time equivalent employee of all U.S. army hospitals using a 90% confidence interval.

Solution: Again, we use a t-based confidence interval. Now, we have $\alpha/2 = .10/2 = .05$ and $n-1 = 1750$ degrees of freedom. We use the largest value of degrees of freedom in the table (120), so $t_{\alpha/2} \approx 1.658$. The 90% confidence interval is $6563 \pm 1.658 \cdot \frac{2484}{\sqrt{1751}}$, i.e.

$$6563 \pm 1.658 \cdot 59 = 6563 \pm 98.42.$$

9. Each year, construction contractors and equipment distributors from across the United States participate in a survey called the CIT Construction Industry Forecast. Recently, 900 contractors were interviewed for the survey. Of these, 414 indicated that they either already have a company website or plan to have a company website by the end of the year.

   (a) Estimate the proportion of contractors in the United States who have a company website or who will have one by the end of the year.

   Solution: We estimate this using the sample proportion:

   $$\hat{p} = \frac{414}{900} = .46.$$

   (b) Find an interval estimate for the proportion, using a 95% confidence interval.

   Solution: We use $n = 900$. The 95% confidence interval is $.46 \pm 2\sqrt{\frac{(.46)(1-.46)}{900}}$, i.e.

   $$.46 \pm 2 \cdot .017 = .46 \pm .034.$$

Standard Errors

10. For each of the problems on this handout, compute the Standard Error of the estimate of the population parameter.

Solution:

- 1(a): $\text{SE}(\bar{x}) = 18.5$
- 1(b): $\text{SE}(\bar{x}) = 1.3$
- 1(c): $\text{SE}(\bar{x}) = 1.00$
- 4(a): $\text{SE}(\hat{p}) = .06$
- 4(b): $\text{SE}(\hat{p}) = .05$
- 4(c): $\text{SE}(\hat{p}) = .06$
- 7: $\text{SE}(\hat{p}) = .014$
- 8: $\text{SE}(\bar{x}) = 59$
- 9: $\text{SE}(\hat{p}) = .017$