Covariance and Correlation

STAT-UB.0103 – Statistics for Business Control and Regression Models

Portfolio Optimization

1. Suppose there are two stocks, X and Y. The annual returns for these stocks can be modeled as independent random variables. Suppose that the expected returns for the two stocks are both equal to 5%, and the standard deviations of the returns for the two stocks are both equal to 1%. Suppose you invest \$30 in stock X and \$70 in stock X.

   (a) If after one year, the return from stock X is 6.0% and the return from stock Y is 4.8%, what is your gain?

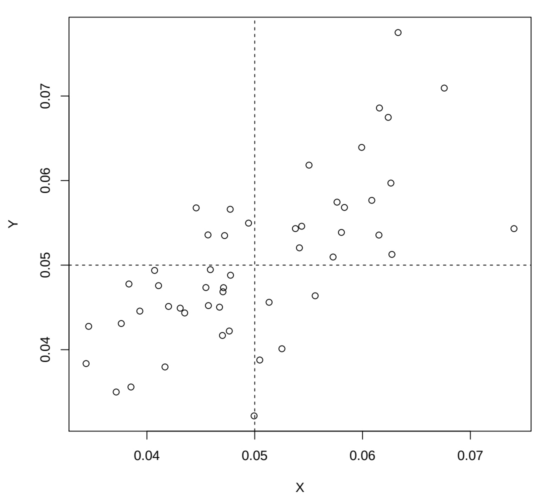
   (b) Suppose you do not know what the returns will be one year from now, and you take them to be random. What is your expected gain?

   (c) Suppose I give you \$100 to invest in stocks X and Y. List some strategies for splitting the money between these two stocks.

(d) What are the expected gains from the strategies you devised in part (c)?

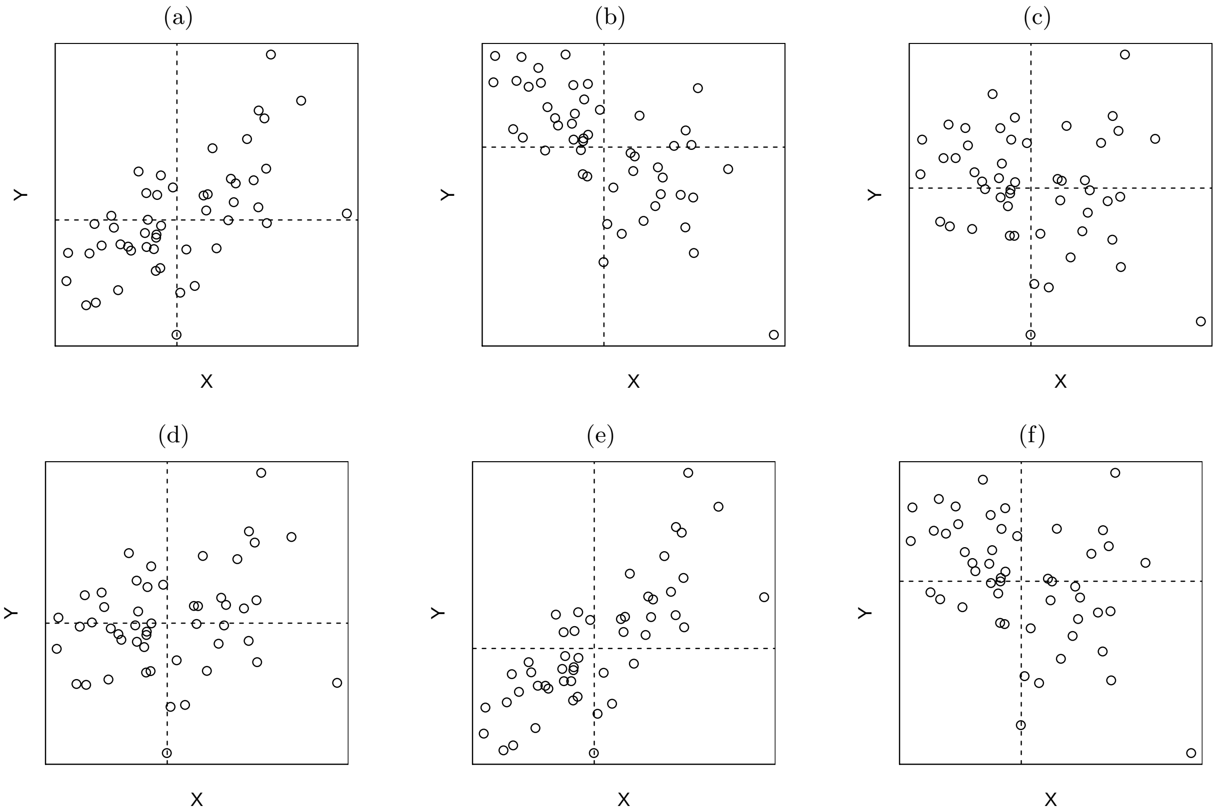
(e) Is there any difference between these investment strategies? Which one should you choose?

(f) Here is a plot of the annual returns for stocks X and Y for the last fifty years. The means for the two stocks are shown by the dashed lines. Does this plot indicate any problems with the assumptions above?



Covariance

2. Parts (a)–(f) show plots of 50 random variable (X, Y) pairs sampled from four different 2-dimensional distributions. Dashed lines indicate the expectations of X and Y. In each part, decide if the covariance between X and Y seems to be positive, negative, or negligible.



3. Suppose X and Y are random variables with $\text{var}(X) = 4$, $\text{var}(Y) = 3$, and $\text{cov}(X, Y) = -2$.
   (a) Find $\text{var}(X + Y)$.

   (b) Find $\text{var}(2X + 5Y)$.

   (c) Find $\text{var}(3X - Y)$.

4. Suppose X and Y are random variables with means $\mu_X = 10$, $\mu_y = 5$, standard deviations $\sigma_X = 2$, $\sigma_Y = 4$, and correlation $\rho_{XY} = -.40$. Find $\text{var}(X + Y)$.

5. Suppose X and Y are random variables with means $\mu_X = -10$, $\mu_y = 3$, standard deviations $\sigma_X = 4$, $\sigma_Y = 1$, and correlation $\rho_{XY} = .50$. Find $\text{var}(X - 2Y)$.