Random variables (review)

1. Let X be a random variable describing the number of cups of coffee a randomly-chosen NYU undergraduate drinks in a week. Suppose that there is a 10% chance that the student has one cup of coffee, 30% chance that the student has two cups of coffee, 40% chance that the student has 3 cups of coffee, and a 20% chance stat the student has four cups of coffee.

   (a) Let $p(x)$ be the probability distribution function of X. Fill in the following table:

x	1	2	3	4
$p(x)$				

   (b) Find $E(X)$, the expectation of X.

   (c) What is the interpretation of the expectation of X?

Variance and Standard Deviation

2. This is a continuation of problem 1.

   (a) Find $var(X)$ and $sd(X)$, the variance and standard deviation of X.

   (b) What is the interpretation of the standard deviation of X?

3. Consider the following game:

    1. You pay $6 to pick a card from a standard 52-card deck.
    2. If the card is a diamond ($\diamondsuit$), you get $22; if the card is a heart ($\heartsuit$), you get $6; otherwise, you get nothing.

   Perform the following calculations to decide whether or not you would play this game.

   (a) Let W be the random variable equal to the amount of money you win from playing the game. If you lose money, W will be negative. Find the PDF of W.

   (b) What are your expected winnings? That is, what is μ, the expectation of W?

   (c) What is the standard deviation of W?

   (d) What are the interpretations of the expectation and standard deviation of W?

Properties of Expectation and Variance

4. **Affine Transformations.** Let X be a random variable with expectation $\mu_X = 2$ and standard deviation $\sigma_X = 3$.

   (a) What is the expectation of $5X + 2$?

   (b) What is the standard deviation of $5X + 2$?

5. **Sums of Independent Random Variables.** Let X and Y be independent random variables with $\mu_X = 1$, $\sigma_X = 3$, $\mu_Y = -5$, $\sigma_Y = 4$.

   (a) What is $E(X + Y)$?

   (b) Find $\text{var}(X + Y)$ and $\text{sd}(X + Y)$.

6. Let X and Y be independent random variables with $\mu_X = -2$, $\sigma_X = 1$, $\mu_Y = 3$, $\sigma_Y = 4$.

   (a) Find the expectation and standard deviation of $-3X + 2$.

   (b) Find the expectation and standard deviation of $X + Y$.

   (c) Find the expectation and standard deviation of $-3X + Y + 2$.

Advanced Problems

7. **Bernoulli random variable.** Suppose you flip a biased coin that lands Heads with probability p and lands tails with probability $1 - p$. Define the random variable

$$X = \begin{cases} 1 & \text{if the coin lands Heads;} \\ 0 & \text{if the coin lands Tails.} \end{cases}$$

This random variable is called a "Bernoulli random variable with success probability p."

(a) What is the PDF of X?

(b) Find μ, the expectation of X

(c) Find σ^2, the variance of X.

8. Suppose you have a biased coin that lands Heads with probability p and lands Tails with probability $1 - p$. You flip the coin 2 times. Let Y be the number of times the coin lands Heads.

(a) What is $E(Y)$?

(b) What is $\text{var}(Y)$?
   Hint: $Y = X_1 + X_2$, where X_1 and X_2 are independent Bernoulli random variables corresponding to the 2 coin flips. Use the answer to problem 7(c).

(c) Suppose instead that you flip the coin n times, and let Y count the number of Heads. What are the expectation and variance of Y?
   Hint: $Y = X_1 + X_2 + \cdots + X_n$.