Intro to Hypothesis Tests – Solutions

STAT-UB.0103 – Statistics for Business Control and Regression Models

Confidence Intervals (Review)

1. Each year, construction contractors and equipment distributors from across the United States participate in a survey called the CIT Construction Industry Forecast. Recently, 900 contractors were interviewed for the survey. Of these, 414 indicated that they either already have a company website or plan to have a company website by the end of the year.

   (a) Estimate the proportion of contractors in the United States who have a company website or who will have one by the end of the year.

   > **Solution:** We estimate this using the sample proportion:
   >
   > $$\hat{p} = \frac{414}{900} = .46.$$

   (b) Find an interval estimate for the proportion, using a 95% confidence interval.

   > **Solution:** We use $n = 900$. The 95% confidence interval is $.46 \pm 2\sqrt{\frac{(.46)(1-.46)}{900}}$, i.e.
   >
   > $$.46 \pm 2 \cdot .017 = .46 \pm .034.$$

2. The *Minneapolis Star Tribune* (August 12, 2008) reported that 73% of Americans say that Starbucks coffee is overpriced. The source of this information was a national telephone survey of 1,000 American adults conducted by Rasmussen Reports. Find and interpret a 95% confidence interval for the population proportion.

   > **Solution:** We use the same formula as in the previous problem, but now $\hat{p} = .73$ and $n = 1000$. Thus,
   >
   > $$\sqrt{\hat{p}(1-\hat{p})/n} = .014.$$
   >
   > The 95% confidence interval is
   >
   > $$.73 \pm .028.$$

Null and alternative hypotheses

3. General Mills claims that there are 16 ounces of cereal in the average box that they manufacture. As a consumer advocate, you want to test this claim. What are the null and alternative hypotheses?

> **Solution:** Let μ be the average fill (in ounces).
>
> $$H_0 : \mu = 16,$$
> $$H_a : \mu < 16.$$

4. The Domino's Pizza closest to NYU advertises that their average delivery time to NYU is at most 20 minutes. Set up a null and alternative hypothesis to test this claim.

> **Solution:** Let μ be the average delivery time (in minutes).
>
> $$H_0 : \mu = 20,$$
> $$H_a : \mu > 20.$$

5. McDonald's claims that their quarter pounders weigh at least 1/4 pound, before cooking, on the average. Set up a null and alternative hypothesis to test this claim.

> **Solution:** Let μ be the average weight before cooking.
>
> $$H_0 : \mu = 1/4$$
> $$H_a : \mu < 1/4.$$

6. Pepsi's soda-dispensing machine is design to fill bottles with exactly 2 liters of their product. Find the null and alternative hypotheses in the following two scenarios:

(a) You want to test if the machine is performing *exactly* according to specification.

Solution: Let μ be the average fill level (in liters).

$$H_0 : \mu = 2$$
$$H_a : \mu \neq 2$$

(b) You want to test if the machine is performing according to specification, according to the consumer's perspective.

Solution: Let μ be the average fill level (in liters).

$$H_0 : \mu = 2$$
$$H_a : \mu < 2$$

7. The average nicotine content of a brand of cigarettes must be less than 0.5 mg for it to qualify as a Low Nicotine brand. The manufacturer of Lucky Strikes Cigarettes claims that it is a Low Nicotine brand. The FDA wants to test this claim. What should the null and alternative hypotheses be?

Solution: Let μ be the average nicotine conent (in mg).

$$H_0 : \mu = 0.5$$
$$H_a : \mu > 0.5$$

Type I and Type II errors

8. Refer to question 3. If in reality the manufactured boxes have exactly 16 ounces of cereal on average, but you claim that they have fewer than 16, what type of error are you making?

    Solution: Type I error.

9. Refer to question 4. If the average delivery time is above 20 minutes, but you do not have conclusive evidence of this fact, what type of error are you making?

    Solution: Type II error.

10. Refer to question 5. If a quarter pounder does in fact have an average weight of 1/4 pound before cooking, but you claim that the average weight is less, what type of error are you making?

    Solution: Type I error.

11. Refer to question 7. If the nicotine content is 0.4mg, but you claim that it has above 0.5mg, what type of error are you making?

    Solution: Type I error.

12. Refer to question 7. If the nicotine content is 0.6mg, but you do not have evidence that the content is above 0.5mg, what type of error are you making?

    Solution: Type II error.

13. For the hypothesis testing scenarios in questions 3–7, which type of error is worse, Type I or Type II?

    Solution:

    This depends on whose point of view we are taking (i.e., either the investigator or the status quo). As the investigator, we are usually obligated to err in favor of the status quo. Thus, for defining "worse," we usually take the perspective of the status quo. In these situations, Type I error is always worse.

Two-tailed tests on the population mean (known variance)

14. Is a soda-dispensing machine performing according to specification? Pepsi's dispensing machine is designed to fill bottles with exactly 2 liters of their product. To test if the machine is performing according to specification, we collect a sample of 100 "2-liter" bottles. The average quantity contained in the sample bottles is $\bar{x} = 1.985$ liters. The (population) standard deviation of the fill is known to be 0.05. Test whether the machine is in control, at the 5% level of significance.

    (a) What are the null and alternative hypotheses?

    Solution: The population we are interested in is all bottles filled by the machine (past and future); the mean fill level of the population is μ, and the standard deviation of the fill level of the population is $\sigma = 0.05$. Our null and alternative hypotheses are as follows:

    $$H_0 : \mu = 2$$
    $$H_a : \mu \neq 2.$$

    The nominal (null) value of the population mean is $\mu_0 = 2$.

    (b) What is the test statistic?

    Solution: Our test statistic is based on the sample of $n = 100$ bottles. Let $\bar{x}$ denote the mean fill level from this sample. Our test statistic is

    $$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{\bar{x} - 2}{(.05)/\sqrt{100}} = \frac{\bar{x} - 2}{(.005)}.$$

    (c) What is the rejection region?

    Solution: Since this is a 2-sided alternative, we reject for large values of $|z|$, specifically when $|z| > z_{\alpha/2}$. Since we are testing at the 5% significance level, we have $\alpha = .05$ and $z_{\alpha/2} = 1.96 \approx 2$. We reject H_0 when

    $$|z| > 2.$$

    (d) What assumptions are you making?

    Solution: We are assuming that the sample bottles were drawn independently without bias from the population.

(e) What is the result of the test?

Solution: The observed value of the test statistic is

$$\frac{1.985 - 2}{.005} = -3.$$

Since this value is in the rejection region, we reject H_0 at significance level 5%.