One-sided alternatives

1. The average nicotine content of a brand of cigarettes must be less than 0.5 mg for it to qualify as a Low Nicotine brand. The manufacturer of Lucky Strikes Cigarettes claims that it is a Low Nicotine brand. To test this claim, the FDA takes a random sample of 20 cigarettes (one pack) of Lucky Strikes. They find an average nicotine content of 0.4 mg, with a sample standard deviation of 0.2 mg. Test the manufacturer's claim, at the 1% level of significance. Assume that the nicotine measurements are normally distributed.

   (a) What are the population and the sample?

   (b) What are the null and alternative hypotheses?

   (c) What is the test statistic?

   (d) What is the rejection region?

   (e) What assumptions are you making?

   (f) What is the result of the test?

2. The manager of a credit card company claims that the mean time to settle disputed charges is 30 days. A regulator is worried that the manager's claim is too optimistic. The regulator examines a random sample of 15 disputed charges, and finds a mean time to settlement of 35.9 days, with a sample standard deviation of 10.2 days. Is there evidence at the 5% level of significance to doubt the manager's claim, assuming that the time to settle disputes is normally distributed?

(a) What are the population and the sample?

(b) What are the null and alternative hypotheses?

(c) What is the test statistic?

(d) What is the rejection region?

(e) What assumptions are you making?

(f) What is the result of the test?

Observed significance levels (p-values)

3. Suppose you want to perform a hypothesis test with a two-sided alternative ($H_0 : \mu = \mu_0$, $H_a : \mu \neq \mu_0$) using a z statistic.

   (a) If the observed test statistic is $z = 1.8$, would a level 5% test reject H_0? Would a level 10% reject H_0?
   Note: $z_{.100} = 1.282$, $z_{.050} = 1.645$, $z_{.025} = 1.960$

   (b) What values of α would a level α test reject H_0 if $z = 1.8$?
   Note: $\Phi(1.8) = .9641$

   (c) For this hypothesis test, what is the p-value corresponding to $z = 1.8$? That is, if H_0 is true and we repeat the random experiment, what is the chance of getting a test statistic at least as extreme as the value $z = 1.8$?

4. Suppose you want to perform a hypothesis test with a one-sided, greater-than alternative ($H_0 : \mu = \mu_0$, $H_a : \mu > \mu_0$) using a z statistic.

   (a) If the observed test statistic is $z = 1.8$, would a level 5% test reject H_0? Would a level 10% reject H_0?
   Note: $z_{.100} = 1.282$, $z_{.050} = 1.645$, $z_{.025} = 1.960$

   (b) What values of α would a level α test reject H_0 if $z = 1.8$?
   Note: $\Phi(1.8) = .9641$

   (c) If H_0 is true and we repeat the random experiment, what is the chance of getting a test statistic at least as extreme as the value $z = 1.8$?

5. In the "Quarter Pounder" example, we we tested the null hypothesis that the weight of a McDonald's quarter pounder is 0.25 pounds ($H_0 : \mu = 0.25$) against the alternative that the weight is below 0.25 pounds ($H_a : \mu < 0.25$). After collecting a sample our observed z statistic was $z = -2.02$. Find the largest level α at which the hypothesis testing procedure does not reject H_0.

6. Suppose we perform a hypothesis test and we observe a p-value of $p = .02$. True or false: There is a 2% chance that the null hypothesis is true.

7. Suppose we perform a hypothesis test and we observe a p-value of $p = .02$. True or false: If we reject the null hypothesis, then there is a 2% chance of making a type I error.

8. Suppose we perform a hypothesis test and we observe a Z test statistic $z = -2.02$, corresponding to a p-value of $p = .02$. True or false: If we were to repeat the experiment and the null hypothesis were actually true, then there would be a 2% chance of observing a test statistic at least as extreme as $z = -2.02$.