Complete Examples

1. Is the crowd wise? National Public Radio’s *Planet Money* podcast performed an experiment to measure the “wisdom of the crowd” with regard to estimating the weight of a cow. After being shown a picture of a cow, respondents were asked to guess its weight, in pounds. The mean of the 17,109 guesses was 1282 pounds, and the standard deviation was 534. The true weight of the cow was 1355 pounds. So, the crowd of 17,109 respondents under-estimated the weight of the cow by 73 pounds. It’s possible that a larger crowd could do better. Given the data available, is this plausible? That is, is it plausible that with a large enough crowd, the estimation error could be made arbitrarily small? We will answer this by performing a hypothesis test.

(a) What is the sample? What is the population? What is the interpretation of the population mean, \( \mu \)?

(b) What are the null and alternative hypotheses, in terms of \( \mu \)?

(c) Compute the test statistic.

(d) How strong is the evidence against the null hypothesis?

(e) Is it plausible that with a large enough crowd, the estimation error could be made arbitrarily small?

2. Before Facebook’s recent redesign, the mean number of ad clicks per day was 100K. In the 49 days after the redesign, the mean number of ad clicks per day was 105K and the standard deviation was 35K. Is there significant evidence that the redesign affected the expected number of ad clicks? Perform a test at the 5% level.

(a) What is the sample? What is the population?

(b) What are the null and alternative hypotheses?

(c) What is the test statistic?

(d) Approximately what is the \( p \)-value?

(e) What assumptions are you making?

(f) What is \( \alpha \)? What is the result of the test?
Types of Errors

3. In a hypothesis test, our decision will either be “reject $H_0$” or “do not reject $H_0$”. Under what situations will each of these decisions be in error?

4. We reject $H_0$ when the $p$-value is below $\alpha$.
   
   (a) If $H_0$ is true, what is the probability of making a Type I error?

   (b) If $H_0$ is false, what is the probability of not making a Type II error?
More $p$-values

5. Suppose we perform a hypothesis test and we observe a $p$-value of $p = .02$. True or false: There is a 2% chance that the null hypothesis is true.

6. Suppose we perform a hypothesis test and we observe a $p$-value of $p = .02$. True or false: If we reject the null hypothesis, then there is a 2% chance of making a type I error.

7. Suppose we perform a hypothesis test and we observe a $T$ test statistic $t = -2.02$, corresponding to a $p$-value of $p = .02$. True or false: If we were to repeat the experiment and the null hypothesis were actually true, then there would be a 2% chance of observing a test statistic at least as extreme as $t = -2.02$. 