Sample Points and Sample Spaces

1. In the following two experiments, what are the sample points and the sample space?
   (a) You flip a coin.
   (b) You roll a 6-sided die.

2. Suppose that a customer visits a restaurant and leaves a review on Yelp with 1–5 stars. What are the sample points and the sample space for the customer’s star rating?

3. Suppose that two customers visit a restaurant, and that they both leave Yelp reviews with 1–5 stars each. What are the sample points and the sample space for the pair of star ratings?

4. Suppose you randomly pick a respondent from the class survey, then record their undergraduate major and gender. What are the sample points and the sample space? Assume that major is either “Business,” “Humanities/Social Science,” or “Science/Engineering.”

Events

5. Suppose that a customer leaves a Yelp rating (1–5 stars) for a restaurant. Describe the event “the rating is odd (not even).”

6. Suppose you randomly pick a respondent from the class survey, then record their undergraduate major and gender. Assume that undergraduate major is listed as “Business,” “Hum./Soc. Sci.,” or “Sci./Eng.,” and that gender is listed as “Male” or ”Female”.
   (a) List the sample points in the event “the major is Business.”
   (b) List the sample points in the event “the gender is Male.”
Probability

7. Suppose you randomly pick a respondent from the class survey and record their undergraduate major and gender.
   (a) Use the following table of recorded survey response frequencies to compute the probabilities of the sample points.

<table>
<thead>
<tr>
<th>Undergrad Major</th>
<th>Gender</th>
<th>Female</th>
<th>Male</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Business</td>
<td></td>
<td>9</td>
<td>6</td>
<td>15</td>
</tr>
<tr>
<td>Hum./Soc. Sci.</td>
<td></td>
<td>10</td>
<td>12</td>
<td>22</td>
</tr>
<tr>
<td>Sci./Eng.</td>
<td></td>
<td>2</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>21</td>
<td>26</td>
<td>47</td>
</tr>
</tbody>
</table>

   (b) Find the probability that the undergraduate major is Business.

   (c) Find the probability that the gender is Male.

   (d) Find the probability the undergraduate major is Humanities/Social Science.

8. Suppose that a customer’s Yelp rating is random, and that the probabilities for the possible star ratings are $p_1 = 10\%$, $p_2 = 30\%$, $p_3 = 25\%$, $p_4 = 20\%$, $p_5 = 15\%$. Find the probability that the rating is odd.
Compound Events and the Additive Rule

9. Suppose you pick a random survey respondent and record their undergraduate major and gender.
   (a) List the sample points in the event “the major is Business or the gender is Male.”

   (b) Compute the probability of the event in part (a) by adding the probabilities of all of the sample points in the event.

   (c) Express the event “the major is Business or the gender is Male” as a union of two other events.

   (d) Compute the probability of the event using the additive rule.

10. Suppose that two customers give ratings (1–5 stars) to the same restaurant on Yelp.
    (a) Express the event “at least one customer gives a 1 star rating” as a union of two other events.

    (b) Suppose that both customers randomly assign their ratings, giving equal probabilities to all possible star ratings. In this case, all 25 sample points have equal probability. Compute the probability of the event in part (a).

11. Suppose that two customers give ratings to the same restaurant on Yelp.
    (a) Express the event “the average of their ratings is 3.5 or 4” as a union of two other events.
        Hint: this is the same event as “the sum of their ratings is 7 or 8.”

    (b) As in problem 10(b), suppose that both customers randomly assign their ratings with equal probability for all possible star ratings, so that all 25 sample points have equal probability. Compute the probability of the event in part (a).
Complementary Events and the Complement Rule

12. Here are the tabulated undergraduate major and gender frequencies from the class survey.

<table>
<thead>
<tr>
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<td>10</td>
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<td>10</td>
</tr>
<tr>
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Use the data and the complement rule to answer the following questions:

(a) If you pick a random survey respondent, what is the probability that the undergraduate major will not be Business?

(b) What proportion of survey respondents have an undergraduate major that is not listed as “Sci./Eng.”?

13. Suppose you flip five coins. What is the probability of getting at least one head?

*Hint: what is the complement of this event?*