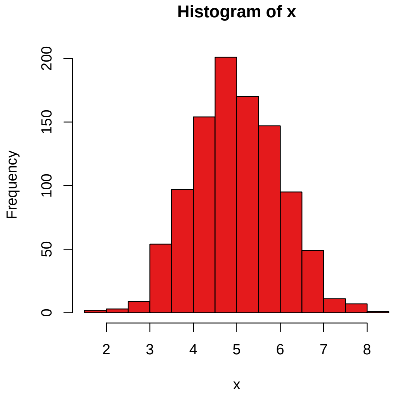
Descriptive Statistics 2 – Solutions

COR1-GB.1305 – Statistics and Data Analysis

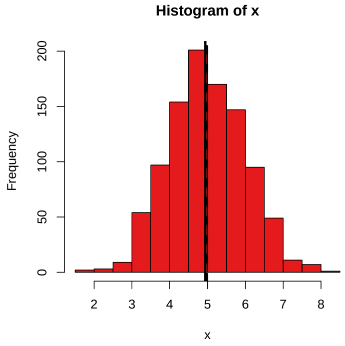
Measures of Central Tendency

1. Here are some histograms. Estimate the mean and median of the data.

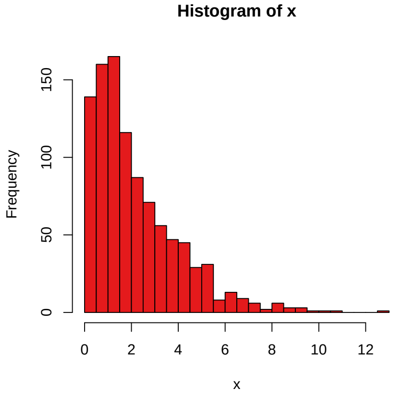
   (a) Symmetric and mound-shaped data.



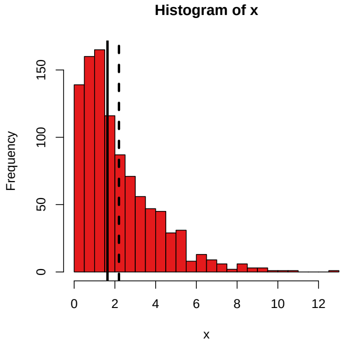
Solution: The median (solid) is roughly in the sample place as the mean (dashed).



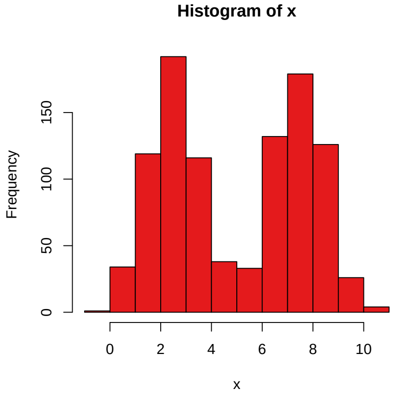
   (b) Skewed data.



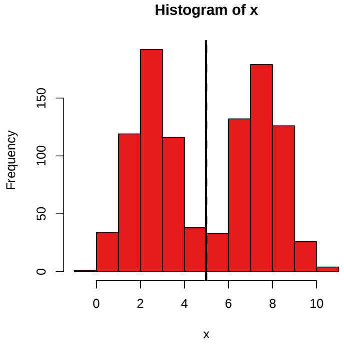
Solution: The mean is pulled to the right by the long tail.



(c) Bimodal data.



Solution: The median and the mean are roughly in the center. Note that neither number conveys much information about the distribution.



2. For the examples (a)–(c) of the previous problem, which is appropriate, the mean or the median?

Solution: This depends on context. If we care about "average" behavior, then mean is typically more appropriate; if we care about "typical" behavior, then median is typically more appropriate.

(a) Both are appropriate; (b) the median is more appropriate for "typical" behavior; mean is more appropriate for "average" behavior; (c) mean is appropriate for "average"; median is not appropriate.

Standard Deviation and The Empirical Rule

3. Forty-two respondents to the class survey reported their GMAT scores. The mean score was 670, and the standard deviation was 40. What can you say about the range of scores reported? Assume that the distribution of reported scores is symmetric and mound-shaped.

Solution: We can use the empirical rule to make the following statements:

- For approximately 68% respondents, reported score is between 630 and 710.
- For approximately 95% respondents, reported score is between 590 and 750.
- For approximately 99.7% respondents, reported score is between 550 and 810.

In fact the true percentages in those intervals are 69%, 88%, and 98%. When the distribution of the data is symmetric and mound-shaped, the predictions from the empirical rule are usually only accurate for the 68% and 95% intervals.

4. The mean reported interest level in the course (out of 10) was 7.2, and the standard deviation was 1.7.

(a) Complete the following statement with appropriate values for X and Y: "Approximately 95% of the survey respondents have interest levels between X and Y."

Solution: $X = 7.2 - 2 \times 1.7 = 3.8$; $Y = 7.2 + 2 \times 1.7 = 10.6$. Of course, it's impossible to have an interest level above 10, so we could also say $Y = 10$.

(b) What assumptions do you need to make for the statement in (a) to be correct? Do you think these assumptions are plausible? How could you check this?

Solution: That the distribution of commute times is symmetric and mound-shaped. We could check this with a histogram. In fact, the histogram is asymmetrical, but even so, there is reasonable agreement with the empirical rule: 69% of interest levels were within 1 standard deviation of the mean; 95% of interest levels were within 2 standard deviations of the mean; 100% of interest levels were within 3 standard deviations of the mean.

(c) What can we do if the assumptions needed in part (b) are not satisfied?

Solution: Sometimes, we can transform the data (e.g., by taking logarithms) to get a variable that has a symmetric, mound-shaped histogram. (For the interest levels, taking logarithms doesn't fix the symmetry/mound-shaped assumptions.)

z-scores

5. Your company has an annual profit of \$60MM with a standard deviation of \$5MM. Assume that the distribution of your annual profits is symmetric and mound-shaped.

   (a) Would it be unusual for your company to have an annual profit of \$52MM?

   Solution: No; 95% of the time, profits are between \$50MM and \$70MM.

   (b) Would it be unusual for your company to have an annual profit of \$83MM?

   Solution: Yes; this would happen less than 99.7% of the time.

6. Fifty respondents from the class survey reported the number of websites they visit on a daily basis. The histogram of these responses was approximately bell-shaped. The mean and standard deviation was was $\bar{x} = 11$ and $s = 7$. How many standard deviations above or below the mean are the following values?

   (a) Visiting 100 websites per day.

   Solution: Let $x_1 = 100$ and let z_1 be the number of standard deviations above of below the mean. Then,

   $$x_1 = \bar{x} + sz_1,$$

   so

   $$z_1 = \frac{x_1 - \bar{x}}{s} = \frac{100 - 11}{7} = 12.7.$$

   Thus, x_1 is 12.7 standard deviations above the mean.

   (b) Visiting 2 websites per day.

   Solution: Let $x_2 = 2$. Then,

   $$z_2 = \frac{x_2 - \bar{x}}{s} = \frac{2 - 11}{7} = -1.3.$$

   Thus, x_2 is 1.3 standard deviations below the mean.

   (c) Visiting 30 websites per day.

   Solution: Let $x_3 = 30$. Then,

   $$z_3 = \frac{x_3 - \bar{x}}{s} = \frac{30 - 11}{7} = 2.7.$$

   Thus, x_3 is 2.7 standard deviations above the mean.

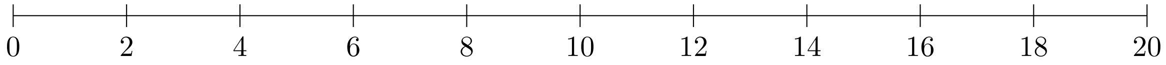
7. In the previous problem, which of the values are unusual?

Solution: The value $x_1 = 100$ is extremely unusual, since this is 12.7 standard deviations away from the mean. The value $x_2 = 30$ might be considered unusual since it is 2.7 standard deviations away from the mean. Typical values are within 2 or 3 standard deviations of the mean (here, "typical" means 95% or 99.7% of the time).

Boxplots

8. Here are the 26 reported answers to the question "How many times do you go out to dinner in a typical month" for the female respondents. The quartiles are shown in bold. Make a boxplot of the data.

1.5, 2, 2, 3, 4, 4, 4, 5, 5, 5, 6, 6, **6**, **6**, 8, 8, 8, 8, 8, 8, 8, 10, 10, 12, 12, 12



9. Here are the answers for the 29 male survey respondents. The middle value is shown in bold. Make a boxplot of the data.

2, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 8, 8, **8**, 8, 8, 8, 8, 9, 9, 10, 10, 10, 10, 12, 12, 15, 20