Regression Inference and Forecasting – Solutions
COR1-GB.1305 – Statistics and Data Analysis

Inference

1. Here are the least squares estimates from the fitting the model

$$\text{Price} = \beta_0 + \beta_1 \text{Size} + \varepsilon_1$$

for $n = 18$ apartments in Greenwich Village. Price is measured in units of \$1000 and size is measured in units of 100 ft^2.

```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
101.375  86.87%     86.05%      81.13%

Coefficients

Term             Coef  SE Coef  T-Value  P-Value   VIF
Constant        182.3     62.4     2.92    0.010
Size(100sqft)   44.95     4.37    10.29    0.000  1.00

Regression Equation

Price($1000) = 182.3 + 44.95 Size(100sqft)
```

(a) Construct a 95% confidence interval for β_1.

> **Solution:** We use
> $$\hat{\beta}_1 \pm t_{\alpha/2}\text{SE}(\hat{\beta}_1),$$
> where $\alpha = .05$ and we have $n - 2 = 16$ degrees of freedom. This gives
> $$44.95 \pm 2.120(4.37) = 44.95 \pm 9.26,$$
> or $(35.69, 54.21)$.

(b) What is the meaning of the confidence interval for β_1?

> **Solution:** We are 95% confident that if we increase size by 100 square feet, then mean price will increase by an amount between \$35.7K and \$54.2K.

(c) What is the meaning of a 95% confidence interval for β_0? In the context of the housing data, is this useful?

> **Solution:** This would be a confidence interval for the mean price of apartments with size 0. This is nonsensical (no apartments have size 0), and thus not useful.

(d) Perform a hypothesis test at level 5% of whether or not the is a linear relationship between Size and mean Price.

Solution: We are interested in the following null and alternative hypotheses:

$$H_0 : \beta_1 = 0 \quad \text{(no linear relationship)}$$
$$H_a : \beta_1 \neq 0 \quad \text{(linear relationship)}$$

Based on the Minitab output, the p-value for this test is below 0.001. Thus, we reject the null hypothesis at level 5%. There is a statistically significant linear relationship between size and mean price.

We can also do this problem using a rejection region. We reject H_0 at level α if $|T| > t_{\alpha/2}$, where

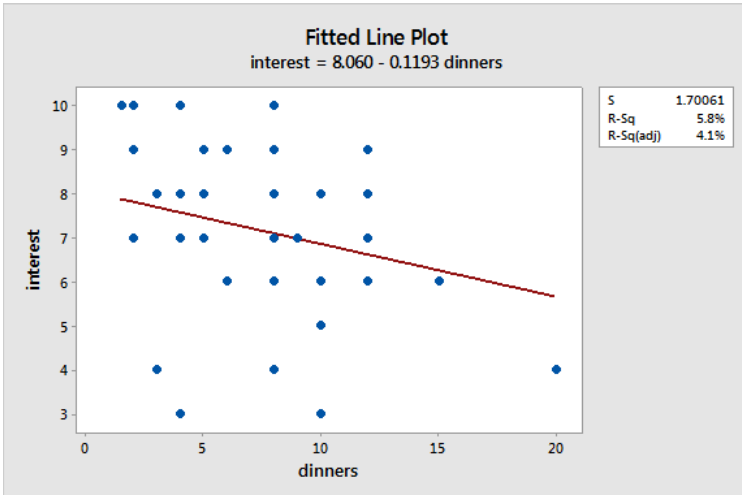
$$T = \frac{\hat{\beta}_1 - 0}{\text{SE}(\hat{\beta}_1)} = \frac{44.95}{4.37} = 10.286$$

For level $\alpha = .05$, we have $t_{\alpha/2} = t_{.025} = 2.120$ (using $n - 2 = 16$ degrees of freedom). Since $|T| > 2.120$, we reject H_0.

2. 54 students reported their interest levels in the course (1–10) and number of times they go out to dinner in a typical month. We will use this data to examine the relationship between these two variables. We fit the model

$$\text{Interest} = \beta_0 + \beta_1 \text{Dinners} + \varepsilon$$

using least-squares. The scatterplot at Minitab regression output follow.




```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
1.70061   5.85%      4.07%       0.00%

Coefficients

Term          Coef  SE Coef  T-Value  P-Value   VIF
Constant     8.060    0.527    15.31    0.000
dinners    -0.1193   0.0658    -1.81    0.075  1.00

Regression Equation

interest = 8.060 -0.1193 dinners
```

(a) Quantify the relationship between Interest and Dinners using a 95% confidence interval. (You will need the value $t_{.025,52} \approx 2.009$.)

Solution: A 95% confidence interval for β_1 is

$$\begin{aligned} \hat{\beta}_1 \pm t_{.025,n-2}\text{se}(\hat{\beta}_1) &= (-0.1193) \pm (2.009)(0.0658) \\ &= -0.1193 \pm 0.1322 \\ &= (-0.2515, 0.0129) \end{aligned}$$

We can be 95% confident that increasing dinners eaten out per month by 1 is associated with changing expected interest in the course by an amount between -0.2515 and 0.0129.

(b) Perform a hypothesis test to determine if there is a significant linear relationship between Interest and Dinners.

Solution: The null and alternative hypotheses are

$$H_0 : \beta_1 = 0 \qquad \text{(no linear relationship)}$$
$$H_a : \beta_1 \neq 0 \qquad \text{(linear relationship)}$$

The p-value for this test is $p = 0.075$. Since $p \geq 0.05$, we do not reject H_0; there is no significant relationship between Interest and Dinners.

Forecasting

3. We used the regression model fit to the housing data to predict price at size 2000 ft^2:

```
Regression Equation

Price($1000) = 182.3 + 44.95 Size(100sqft)


Variable        Setting
Size(100sqft)        20


    Fit   SE Fit        95% CI               95% PI
1081.27  38.1287  (1000.44, 1162.10)  (851.667, 1310.88)
```

(a) Find a 95% confidence interval for the mean price of all apartments with size 2000 ft^2.

> **Solution:** This is given in the output: (1000.4, 1162.1). We 95% confidence, the mean price of all apartments with size 2000 ft^2 is between \$1,000,400 and \$1,152,100.

(b) Find a 95% prediction interval for the price of a particular apartments with size 2000 ft^2.

> **Solution:** Again, this is given in the output: (851.7, 1310.9). If someone tells us that a particular apartment has size 2000 ft^2, then we can say with 95% confidence that the price of the apartment is between \$851,700 and \$1,310,900.

(c) Make a statement about the prices of 95% of all apartments with size 2000 ft^2.

> **Solution:** To make a statement about *all* apartments, we use a prediction interval. With 95% confidence, 95% of all apartments with size 2000 ft^2 have sizes between \$851,700 and \$1,310,900.

(d) What is the difference between the confidence interval and the prediction interval?

> **Solution:** A confidence interval is a statement about the mean value of Y; a prediction interval is a statement about a particular value of Y (equivalently, all values of Y).

4. We fit a regression model to the 294 restaurants from the 2003 Zagat data. Our predictor variable is food quality (1–30), and our response variable is price ($). Here is the result of using the fitted model to predict the price when the food quality is 25.

```
Model Summary

      S    R-sq  R-sq(adj)  R-sq(pred)
12.5559  27.93%     27.68%      26.86%

Coefficients

Term       Coef  SE Coef  T-Value  P-Value   VIF
Constant  -4.74     3.95    -1.20    0.232
Food      2.129    0.200    10.64    0.000  1.00

Regression Equation

Price = -4.74 + 2.129 Food

Variable  Setting
Food           25

    Fit   SE Fit        95% CI              95% PI
48.4832  1.33906  (45.8478, 51.1187)  (23.6315, 73.3349)
```

(a) What is the interpretation of the 95% confidence interval?

> **Solution:** We are 95% confident that the average price of all 2003 New York City restaurants with quality ratings of 25 is between $45.84 and $51.12.

(b) What is the interpretation of the 95% prediction interval?

> **Solution:** Approximately 95% of all 2003 New York City restaurants with quality ratings of 25 have prices between $23.63 and $73.34.

(c) Explain how the confidence interval is related to `Fit`, `SE Fit`, and `S`.

> **Solution:** The 95% confidence interval for $\mathrm{E}(Y \mid x = 25)$ is approximately equal to
>
> $$\hat{y}(25) \pm 2\mathrm{se}(\hat{y}(25)) = 48.4832 \pm (2)(1.33906).$$
>
> (For an exact equivalence, use $t_{.025,n-2}$ instead of 2.)

(d) Explain how the prediction interval is related to `Fit`, `SE Fit`, and `S`.

> **Solution:** The 95% prediction interval is approximately equal to
>
> $$\hat{y}(25) \pm 2s = 48.4832 \pm (2)(12.5559).$$

(For an exact equivalence you would use the formula

$$\hat{y}(x) \pm t_{.025,n-2}\sqrt{s^2 + [\mathrm{se}\{\hat{y}(x)\}]^2};$$

you are not expected to know this formula.)