Confidence Intervals & Hypothesis Tests
STAT-UB.0003: Regression and Forecasting Models

Confidence intervals

1. A random sample of 36 measurements was selected from a population with unknown mean μ and known standard deviation $\sigma = 18$. The sample mean is $\bar{x} = 12$. Calculate a 95% confidence interval for μ.

2. With respect to the previous problem, which of the following statements are true:
   (a) There is a 95% chance that μ is between 6 and 18.
   (b) The population mean μ will be between 6 and 18 about 95% of the time.
   (c) In 95% of all future samples, the sample mean will be between 6 and 18.
   (d) The population mean μ is between 6 and 18.
   (e) None of the above.

3. Complete Problem 1, with a 99% confidence interval instead of a 95% confidence interval.

4. The SoHo Halal Guy at Broadway and Houston currently has 35 Yelp reviews[1] (1 one-star; 1 two-star; 5 three-star; 12 four-star; and 16 five-star). The average star rating is 4.17 and the sample standard deviation of the star ratings is 0.98.

   (a) What is a reasonable population to associate with this sample?

   (b) What is the meaning of the population mean, μ?

   (c) Find a 95% confidence interval for the population mean.

   (d) Under what conditions is the confidence interval valid?

[1] http://www.yelp.com/biz/soho-halal-guy-new-york

Hypothesis tests

5. We collect a simple random sample of size $n = 100$ from a population. The sample mean is $\bar{x} = 12.4$ and the sample standard deviation is $s = 8.0$. Use this data to test the null hypothesis $H_0 : \mu = 10.0$ against the alternative $H_a : \mu \neq 10.0$, where μ denotes the population mean:

   (a) Compute the test statistic.

   (b) Use a z-table to compute an approximate p-value.

   (c) What is the meaning of the p-value? Give a one-sentence description.

   (d) Using a significance level (α) of 5%, what is the result of the hypothesis test?

6. National Public Radio's *Planet Money* podcast performed an experiment to measure the "wisdom of the crowd" with regard to estimating the weight of a cow[2]. After being shown a picture of a cow, respondents were asked to guess its weight, in pounds. The mean of the 17,109 guesses was 1282 pounds, and the standard deviation was 534. The true weight of the cow was 1355 pounds. So, the crowd of 17,109 respondents under-estimated the weight of the cow by 73 pounds. It's possible that a larger crowd could do better. Given the data available, is this plausible? That is, is it plausible that with a large enough crowd, the estimation error could be made arbitrarily small? We will answer this by performing a hypothesis test.

   (a) What is the sample? What is the population? What is the interpretation of the population mean, μ?

   (b) What are the null and alternative hypotheses, in terms of μ?

   (c) Compute the test statistic.

   (d) Use a z-table to compute an approximate p-value; if the test statistic falls outside the z-table, report $p < .0001$.

   (e) In the context of the problem, what is the interpretation of the p-value?

   (f) Is it plausible that with a large enough crowd, the estimation error could be made arbitrarily small?

[2] http://www.npr.org/sections/money/2015/08/07/430372183/episode-644-how-much-does-this-cow-weigh