Confidence Intervals

STAT-UB.0103 – Statistics for Business Control and Regression Models

Population Mean (Known Variance)

1. A random sample of n measurements was selected from a population with unknown mean μ and known standard deviation σ. Calculate a 95% confidence interval for μ for each of the following situations:

   (a) $n = 49$, $\bar{x} = 28$, $\sigma = 28$

   (b) $n = 100$, $\bar{x} = 125$, $\sigma = 50$

   (c) Is the assumption that the underlying population of measurements is normally distributed necessary to ensure the validity of the confidence intervals in parts (a)–(c)?

2. A random sample of 36 measurements was selected from a population with unknown mean μ and known standard deviation $\sigma = 18$. The sample mean is $\bar{x} = 12$. Calculate a 95% confidence interval for μ.

3. With respect to the previous problem, which of the following statements are true:
    A. There is a 95% chance that μ is between 6 and 18.
    B. The population mean μ will be between 6 and 18 about 95% of the time.
    C. In 95% of all future samples, the sample mean will be between 6 and 18.
    D. The population mean μ is between 6 and 18.
    E. None of the above.

4. Complete Problem 2, with a 99% confidence interval instead of a 95% confidence interval.

5. Complete Problem 2, with an 80% confidence interval instead of a 95% confidence interval.

Population Mean (Unknown Variance)

6. A random sample of n measurements was selected from a population with unknown mean μ and unknown standard deviation σ. Calculate a 95% confidence interval for μ for each of the following situations:

   (a) $n = 25$, $\bar{x} = 28$, $s = 12$

   (b) $n = 16$, $\bar{x} = 12$, $s = 18$

   (c) $n = 100$, $\bar{x} = 125$, $s = 50$

   (d) Is the assumption that the underlying population of measurements is normally distributed necessary to ensure the validity of the confidence intervals in parts (a)–(c)?

7. In each of the following situations, find α and $t_{\alpha/2}$.
   (a) An 80% confidence interval with $n = 10$.

   (b) A 99% confidence interval with $n = 25$.

   (c) A 90% confidence interval with $n = 30$.

8. Compute $z_{\alpha/2}$ for each of the situations in problem 7.

9. The white wood material used for the roof of an ancient Japanese temple is imported from Northern Europe. The wooden roof must withstand as much as 100 centimeters of snow in the winter. Architects at Tohoku University (Japan) conducted a study to estimate the mean bending strength of the white wood roof. A sample of 25 pieces of the imported wood were tested and yielded the following statistics on breaking strength: $\bar{x} = 74.5$, $s = 10.9$. Estimate the true mean breaking strength of the white wood with a 90% confidence interval.

10. Researchers recorded expenses per full-time equivalent employee for each in a sample of 1,751 army hospitals. The sample yielded the following summary statistics: $\bar{x} = \$6,563$ and $s = \$2,484$. Estimate the mean expenses per full-time equivalent employee of all U.S. army hospitals using a 90% confidence interval.