Complementary Events / Counting – Solutions
STAT-UB.0103 – Statistics for Business Control and Regression Models

Complementary Events and the Complement Rule

1. Here are the tabulated major and gender frequencies from the class survey.

	Gender		
Major	Female	Male	Total
Finance	12	20	32
Other	4	3	7
Undecided	10	15	25
Total	26	38	64

Use the data to answer the following questions:

(a) If you pick a random survey respondent, what is the probability that the major will not be Finance?

Solution: Let

$$A = \{ \text{ the randomly picked student student } \textit{is not} \text{ a Finance major } \}.$$

Then, the complement of this event is

$$A^c = \{ \text{ the randomly picked student does } \textit{is} \text{ a Finance major } \}.$$

By the complement rule,

$$\begin{aligned} \mathrm{P}(A) &= 1 - \mathrm{P}(A^c) \\ &= 1 - \tfrac{32}{64} \\ &= \tfrac{32}{64} \\ &= 50\%. \end{aligned}$$

(b) What proportion of survey respondents have decided on a major?

Solution: Again, using the complement rule,

$$1 - \tfrac{25}{64} = \tfrac{39}{64} = 61\%.$$

2. Suppose you flip five coins. What is the probability of getting at least one head?
Hint: what is the complement of this event?

Solution: The sample space, Ω, is the set of all possible outcomes for the five flips. Since there are 5 independent flips, and each has 2 possible outcomes, we have that $|\Omega| = 2^5 = 32$.

Let

$$A = \{ \text{ you get at least one head } \}.$$

Then,

$$\begin{aligned} A^c &= \{ \text{ you don't get any heads } \} \\ &= \{(T, T, T, T, T)\}. \end{aligned}$$

Thus, by the complement rule,

$$\begin{aligned} \mathrm{P}(A) &= 1 - \mathrm{P}(A^c) \\ &= 1 - \tfrac{1}{32} \\ &= \tfrac{31}{32}. \end{aligned}$$

The Multiplication Rule

3. A man has 4 pair of pants, 6 shirts, 8 pairs of socks, and 3 pairs of shoes. Ignoring the fact that some of the combinations may look ridiculous, how many ways can he get dressed?

Solution: Using the multiplication rule, there are

$$4 \cdot 6 \cdot 8 \cdot 3 = 576$$

ways for the man to get dressed.

4. A restaurant offers soup or salad to start, and has 11 entrees to choose from, each of which is served with rice, baked potato, or zucchini. How many meals can you have if you can choose to eat one of their 4 desserts or have no desert?

Solution:

$$2 \cdot 11 \cdot 3 \cdot 5 = 330$$

Note that there are 5 choices for the final course (4 desserts or no dessert).

5. How many answer sheets are possible for a true/false test with 15 questions?

Solution:

$$2^{15} = 32768$$

Permutations

6. How many ways can 5 people stand in line?

Solution:

$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5! = 120$$

7. How many different batting orders are possible for 9 baseball players?

Solution:

$$9! = 362880$$

8. How many ways can 8 books be put on a shelf?

Solution:

$$8! = 40320$$

More Permutations

9. Twelve people belong to a club. How many ways can they pick a president, vice-president, secretary, and treasurer?

Solution:

$$12 \cdot 11 \cdot 10 \cdot 9 = \frac{12!}{8!} = 11880$$

10. In a horse race the first three finishers are said to win, place, and show. How many finishes are possible for a race with 11 horses?

Solution:

$$11 \cdot 10 \cdot 9 = \frac{11!}{8!} = 990$$

11. Five different awards are to be given to a class of 30 students. How many ways can this be done if (a) each student can receive any number of awards, (b) each sutent can receive at most one award?

Solution: (a) $30^5 = 24300000$
(b) $30!/(25!) = 30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 = 17100720$

Combinations

12. A club has 23 members.
    (a) How many ways can they pick 2 people to be on a committee to plan a party?

Solution:

$$\binom{23}{2} = \frac{23 \cdot 22}{2 \cdot 1} = 253.$$

(b) How many ways can they pick 4 people to be on a committee to plan a party?

Solution:

$$\binom{23}{4} = \frac{23 \cdot 22 \cdot 21 \cdot 20}{4 \cdot 3 \cdot 2 \cdot 1} = 8855.$$

13. A restaurant offers 15 possible toppings for its pizza. How many different pizzas with 3 toppings can be ordered?

Solution:

$$\binom{15}{3} = \frac{15 \cdot 14 \cdot 13}{3 \cdot 2 \cdot 1} = 455$$

Advanced Problems

14. **New York state lotto.** You pick six of the numbers 1 through 54, and then in a televised drawing six of the numbers are selected. If all six of your numbers are selected then you win a share of the first place prize. If five or four of your numbers are selected you win a share of the second or third prize.

    (a) How many ways are there to select 6 numbers for the lotto ticket?

    Solution:
    $$\binom{54}{6} = \frac{54 \cdot 53 \cdot 52 \cdot 51 \cdot 50 \cdot 49}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 25827165$$

    (b) How many ways are there to select a first prize number?

    Solution:
    $$1$$

    (c) What is the probability of selecting a first prize number?

    Solution:
    $$\begin{aligned} P(\text{first prize}) &= \frac{\#\{\text{lotto tickets that match all six numbers}\}}{\#\{\text{lotto tickets}\}} \\ &= \frac{1}{\binom{54}{6}} \\ &= 1/25827165 \\ &= 0.000004\% \end{aligned}$$

15. **Quality assurance.** Suppose we have a batch of 100 light bulbs, which contains 5 defective bulbs. If we pick 10 for testing, what is the probability that no bulbs in the sample are defective? We can answer this question in three steps.

    (a) How many ways are there of picking 10 bulbs for testing out of 100?

    Solution:
    $$\binom{100}{10}$$

(b) How many ways are there of picking 10 non-defective bulbs?

Solution:

$$\binom{95}{10}$$

(c) What is the probability that there are no defective bulbs in your sample of 10?

Solution:

$$\begin{aligned} P(\text{no defects in sample}) &= \frac{\binom{95}{10}}{\binom{100}{10}} \\ &= 58\%. \end{aligned}$$