# $p$-values / Tests on Proportions

STAT-UB.0103 – Statistics for Business Control and Regression Models

## $p$-values

1. In the "Quarter Pounder" example, we we tested the null hypothesis that the weight of a McDonald's quarter pounder is 0.25 pounds ($H_0 : \mu = 0.25$) against the alternative that the weight is below 0.25 pounds ($H_a : \mu < 0.25$). After collecting a sample our observed $z$ statistic was $z = -2.02$. Find the largest level $\alpha$ at which the hypothesis testing procedure does not reject $H_0$.

2. Suppose we perform a hypothesis test and we observe a $p$-value of $p = .02$. True or false: There is a 2% chance that the null hypothesis is true.

3. Suppose we perform a hypothesis test and we observe a $p$-value of $p = .02$. True or false: If we reject the null hypothesis, then there is a 2% chance of making a type I error.

4. Suppose we perform a hypothesis test and we observe a $Z$ test statistic $z = -2.02$, corresponding to a $p$-value of $p = .02$. True or false: If we were to repeat the experiment and the null hypothesis were actually true, then there would be a 2% chance of observing a test statistic at least as extreme as $z = -2.02$.

## Tests on a population proportion

5. Suppose you have a population with an unknown proportion $p$ of successes. You want to test the null hypothesis $H_0 : p = 0.2$ against the alternative $H_a : p \neq 0.2$. To this end, you collect a sample of size $n = 100$. It terns out that there are $x = 30$ successes in the sample, so that the sample proportion is $\hat{p} = \frac{30}{100} = .30$. Is there sufficient evidence to reject the null hypothesis at level 5%?

6. Suppose you have a population with an unknown proportion $p$ of successes. You want to test the null hypothesis $H_0 : p = 0.6$ against the alternative $H_a : p > 0.6$. To this end, you collect a sample of size $n = 81$ It terns out that there are $x = 53$ successes in the sample, so that the sample proportion is $\hat{p} = \frac{53}{81} = .654$. You want to perform a hypothesis test at level 5%.
   (a) What is the test statistic?

   (b) What is the rejection region?

   (c) What is the result of the hypothesis test?

7. In a May 2006 random-digit-dialiing telephone survey of 4,000 American adults, 42% of the sample had access to a high-speed internet connection at home. Let $p$ represent the true proportion of all American adults who had access to a high-speed internet connection at home in 2006.

   In 2005, the Pew Internet & American Life Project reported that 30% of all American adults had access to a high-speed internet connection.

   Perform a test at significance level 5% of whether the proportion changed in 2006.

   (a) What are the population and the sample?

   (b) What are the null and alternative hypotheses?

   (c) What is the test statistic?

   (d) What is the rejection region?

   (e) What assumptions are you making?

   (f) What is the result of the test?

8. An alkaline battery manufacturer wants to be reasonably certain that fewer than 5% of its batteries are defective. Suppose 300 batteries are randomly selected from a very large shipment; each is tested, and 10 defective batteries are found. Does this provide sufficient evidence for the manufacturer to conclude that the fraction defective in the entire shipment is less than .05? Use a significance level of $\alpha = .01$.

   (a) What are the population and the sample?

   (b) To err on the side of caution, the manufacturer takes the null hypothesis to be that the shipment has a high proportion of defective batteries. In terms of the population parameter, what are the null and alternative hypotheses?

   (c) What is the test statistic?

   (d) What is the rejection region?

   (e) What assumptions are you making?

   (f) What is the result of the test? Is there sufficient evidence to conclude that the defect rate is acceptable?