The Birthday Problem

1. A class has 70 students. What is the probability that at least two students have the same birthday? Assume that each person in the class was assigned a random birthday between January 1 and December 31.

Independence

2. Suppose you flip two fair coins. Let $A =$ "the first coin shows Heads," $B =$ "The second coin shows Heads." Find the probability of getting Heads on both coins, i.e. find $P(A \cap B)$.

3. Suppose you roll three dice. What is the probability of getting exactly one 6?

4. Suppose you roll two dice, one red and one green. Let A = "The sum is 7," B = "The red die is a 6." Are events A and B independent?

5. Suppose you roll two dice, one red and one green. Let A = "The sum is 8," B = "The red die is a 6." Are events A and B independent?

6. Suppose you have a database of 300K reviews from 15K businesses and 70K users. In each of the following scenarios, you randomly sample 2 reviews. Define events A and B as

$$A = \text{the first review is 4 or 5 stars}$$
$$B = \text{the second review is 4 or 5 stars}$$

In which sampling schemes are events A and B independent? Assume that all samples are random and unbiased. Explain your answers.

(a) You sample two distinct reviews from the entire dataset.

(b) You randomly sample one business from the dataset, then sample two distinct reviews of the business.

(c) You randomly sample one user from the dataset, then sample two distinct reviews written by the user.

(d) You randomly sample two distinct users from the dataset, then sample one review written by each user.

(e) You randomly sample two distinct businesses from the dataset, then sample one review from each business.